Discovering Probabilities	
Respective blueprint	Knucklebone Dice
Description	After creating the Knucklebone Dice with a 3D printer, introduce the notions of probability through play.
Learning Objectives	1: Discovery of the Knucklebone Dice 2: Probabilities and the game 3: Calculus of probabilities
Related curricular subject(s)	Mathematics (probabilities
Prerequisites / preparatory actions for teachers	Print the Knucklebone Dice
Prerequisites / preparatory actions for students	Notion of probabilities
Age of students	Starting from 10

Duration	1h
Level of difficulty	Medium

Step by step description of the tasks

Step 1 Discovery of the Knucklebone Dice

The teacher explains what this game is. A knucklebone is a small bone. The talus is a joint bone found in the tarsus of the carpus (wrist skeleton) of the hind legs of sheep or goats.

In Antiquity, the Greeks and Romans used these knucklebones for dice games, trying to achieve certain combinations.

A knucklebone has four different faces:

- Of the two wider sides, one is convex, the other concave.

- Of the two narrower faces, one is flat, the other sinuous.

The faces were assigned values and bets were made on the sums of the numbers of the resulting faces:

- The flat face "planum" is worth 1,
- The concave face (hollow) "supinum" is worth 3,
- The convex face (bump) "pronum" is worth 4,

The sinuous face "tortuosum" is worth 6.

The numbers 2 and 5 are not used.

The modern game is quite different. It consists of 5 metal or plastic knuckles, one of which is painted red, called the father.

Step 2 Proba and the game

Note that, as with the current cubic dice, the sum of the values of the opposite sides is always 7. Moreover, the choice of the values of the faces was based on the observation that narrow faces come out less often than wide faces; when throwing a Knucklebone, the numbers 1 and 6 appear less frequently than the numbers 3 and 4.

Historical rules of the game

In the old game, four knucklebones were usually thrown simultaneously, or one after the other.

Named *Venus* by the Romans, the most favourable configuration of knucklebone was when all four knucklebones were on different sides; in the worst, named *Canis* (Latin for dog), they were all on the narrow flat side (the vulture side).

The teacher sets up groups of 3 or 4 students.

The students are given 30 tokens and take turns to make 100 successive throws.

Game 1

- The students take turns throwing four knucklebones simultaneously

- If he gets a *Venus* shot, the player takes the common pot

- If not, he/she pours into the common pot the equivalent of the sum of the numbers equivalent to the faces obtained (Tortuosum = 6, Pronum = 4, Supinum = 3, Planum = 1)

The results of the 100 successive throws are then summarised by noting:

- The number of games played according to the type of face obtained

- The average number of throws to get a *Venus* shot

Students discuss and analyse their results.

<u>Step 3 Calculus of probabilities</u>

The Knucklebone is an object that is not naturally symmetrical. If you throw it and note the face you get, the events of *getting one of the four*

faces of the knucklebone in this random experiment are not equiprobable.

The experiment shows that if p1, p2, p3, and p4 are the probabilities of obtaining, after throwing, the faces Pronum, Supinum, Planum, and Tortuosum respectively, then:

p1 = p2

p3 = p4

p1 = 4p3.

The flat face and the sinuous face are 4 times less likely to appear as a result after a throw than the wide faces.

The teacher asks the question

What can be said about the sum

p1 + p2 + p3 + p4?

Deduce the values of p1, p2, p3, p4.

>> A knucklebone is thrown. For a first throw, the universe of possibilities is Ω = {1, 2, 3, 4}.

The probability law, Σpi = 1, allows us to write:

p1 + p2 + p3 + p4 = 1

Using the experimental results above, we find:

2p1 + 2p3 = 1

2p1 + 2 (p1/4) = 1

Therefore,

p1 = p2 = 2/5

p3 = p4 = 1/10

Mathematical questions that can be asked of students

a) How many possibilities do we have after a single throw of a single knucklebone?

After one throw, we have a 2/5 probability of getting the Pronum and Supinum faces, so a 4 or a 3, and 1/10 the Planum and Tortuosum faces, so a 6 or a 1

To go further: play by rolling 4 dice !

a) Consider the event A: obtaining an odd number

What is the probability of this event?

b) Consider event B: get a supinum or a pronum. What is the probability of this event?

Assessment activities

The following questions can be asked by teachers to evaluate the activity:

What is the probability law?

Are the events of *getting one of the four faces* of the knucklebone in this experiment equiprobable?