Pythagorean tuning	
Respective blueprint	Monochord
Description	Discovery of fractions and musical scales through the work of Pythagoras
Learning Objectives	1: Discovering Pythagoras 2: Discovering the ratio of harmonious musical notes 3: Using fractions and musical notes
Related curricular subject(s)	Fractions History of mathematics
Prerequisites / preparatory actions for teachers	Create the monochord (using the corresponding blue Have a working notion of musical scales
Prerequisites / preparatory actions for students	None

Age of students	12-15
Duration	1h
Level of difficulty	Medium

Step by step description of the tasks

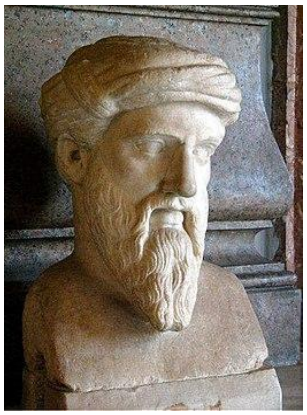
Step 1. Discovering Pythagoras

The teacher introduces the mathematician to the whole class.

Who knows about Pythagoras?

Little is known about the life and work of Pythagoras. He was probably born in the sixth century B.C., on Samos, an island of the Aegean Sea. After long journeys in which he learnt much, he would have gone into exile in Crotone, in southern Italy. He founded a school, which imposed strict life rules and took the form of an influential brotherhood.

Pythagoreanism was a philosophical movement, religious and moral, but also political. Eventually, it is said that Cylon, one of Pythagoras's opponents, led a revolution against him, which brought about the end of the school and scattered Pythagoras's followers. Very early in ancient history, Pythagoras became a legendary figure.



Credit Wikipedia

The Pythagoreans were interested in the ratio of notes from the 6th century BC. They used a monochord to vary the length of a string and found that the shorter the string, the higher the sound.

The Pythagoreans, but also all the Greeks, had a very precise idea of numbers. Only the whole number was considered a number and the simpler the numbers, the greater the beauty.

Hence the theoretical construction of dividing the string of monochord into 2, 3 and 4, the smallest numbers.

Step 2: Discovering the ratio of harmonious musical notes

The teacher explains the ratio between the harmonious notes.

In the way of the school of Pythagoras, let's find the ratio of harmonious musical notes!

For this step, it would be ideal to set up an interdisciplinary lesson with a music teacher colleague. He/she could provide answers to the following questions, for example:

Why do we say that we have "the same note" by dividing the string in two?

Why are sounds in fifths and fourths harmonious to our Western ears?

Credit Wikipedia

Pythagoras would have weighed the 3 metal blocks and found a ratio of the weights to the sound. Let's imagine:

Block 1: 1kg

Block 2: 2kg

Block 3: 3kg

He would have realized that the sound is harmonious when the sound of a block and the sound of a block that weighs twice as much sound together...

Find the ratio between the block 1 and the block 2

> Block 1 + Block 2: ratio ½

He would also have realized that the sound is harmonious when the sound of the first block and the sound of the third block sound together...

Find the ratio between the block 1 and the block 3

> Block 1 + Block 3: ratio 1/3

This ratio can be transcribed to a length, and therefore to a string length [> monochord], and by listening to the sound produced we can name it with the name of a note.

Step 3: Using fractions and musical notes

The teacher can now present the painting entitled The School of Athens, which Raphael painted to the glory of rational thought, and which brings together the great names of Greek philosophy.

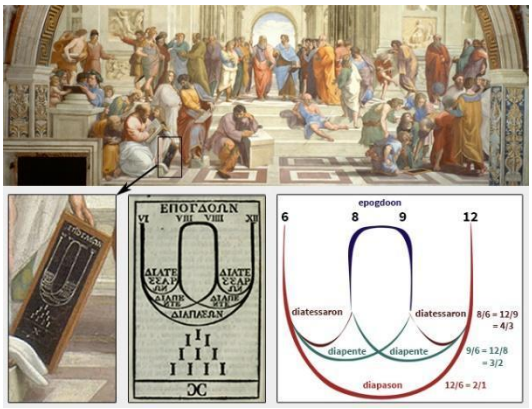
Credit Wikipedia

Pythagoras is represented with disciplines, and one of them holds a tablet on which are engraved the two symbols of Pythagoreanism chosen by the painter:

-the tetraktys, which groups together the first four integers, which add up to the number 10 (10 =1+2+3+4)

- the "musical proportion" in which are grouped the fundamental consonances born of the three classical averages: arithmetic, geometric and harmonic.

Credit Utpictura18



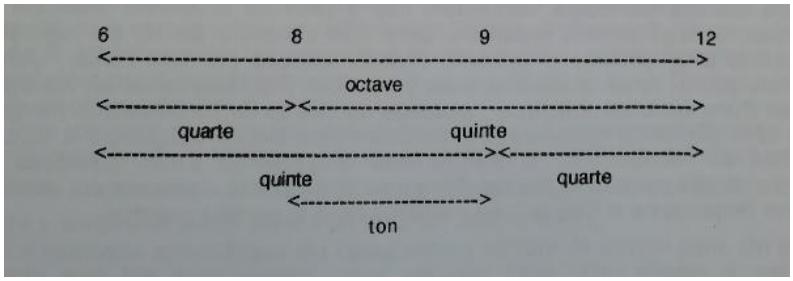
Credit https://edutheque.philharmoniedeparis.fr

If we stretch a string of a given length taken as a unit and pluck it: it makes a certain sound.

If we take the half, then the 2/3, then the 3/4, we obtain different sounds defined respectively as: the upper octave (diapason), the fifth (diapente) and the fourth (diatessaron).

By representing the total length by 12, the smallest common multiple of 3 and 4, the following table illustrates the "musical proportion" which,

according to the Neo-Pythagorean Nicomachus of Gerasus, is "the most useful for all progress in music and in the knowledge of nature".



Credit M. Spiesser

This table highlights the division of the octave into fifths + fourths or fourths + fifths and shows that the additive language of intervals is transformed into a multiplicative language when it comes to fractions since 2/3 x 3/4 = 1/2 (fifths + fourths = octave).

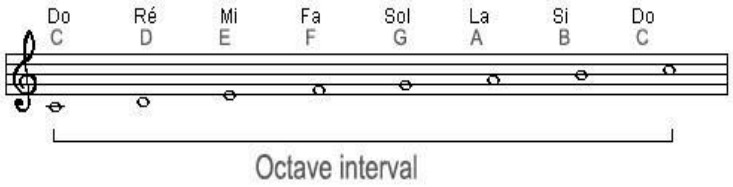
B/ Test it with an instrument!

The teacher introduces the monochord, explains how they made it, and involves different students in the rest of this step

On the monochord, if the string is sounded without anything, the note produced is low C (Do).

if a bridge is placed in the middle of the string, the sound produced by plucking the half-string is identical to the sound produced by the whole string, but it is higher (one octave higher = high C (Do)). The octave corresponds to the ratio of 2 to 1.

The octave is the interval between the low C (Do) and the high C (Do) above it.



The teacher distributes a string to the students (each student receives a 50cm string).

Take your string and find its middle.

Let's hear it on the monochord

>Understand that the fraction 1/2 is equal to the position of the bridge and the strings stretched on each side gives the sound C (high).

Low C = 2/2

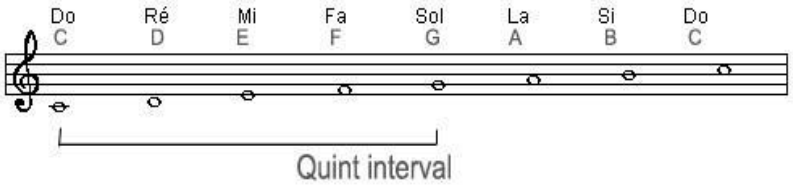
High C = ½

This ratio is called an octave.

With this instrument, Pythagoras would also have been able to calculate mathematically the other sound ratios of musical intervals.

For example, the quint with a ratio of 3 to 2.

Indeed, if you position the bridge at 1/3 of the string, and if we sound 2/3 of the string it corresponds to the sound of the note G. The interval from C to G is called a quint. The quint of C is G.



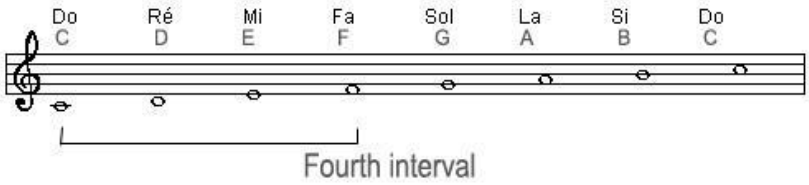
Take your string and find and bend the string in 3

Let's hear it on the monochord

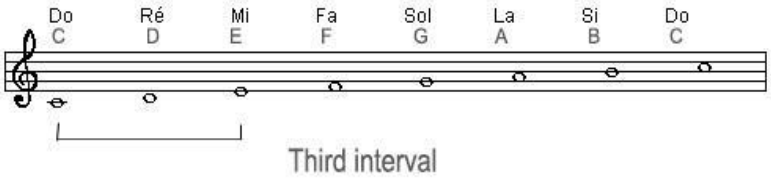
>Understand the fraction 1/3; 2/3 and therefore 3/3

Other examples:

The interval between C and F is called a fourth. Location ¼.



The interval between C and E is called a third. Location 1/5.



The teacher can play the examples on the monochord.

Assessment activities

The following questions can be asked by teachers to evaluate the activity:

In which country was Pythagoras born?

What is the ratio of 1/2 in musical notes called?