Measure the height of your school

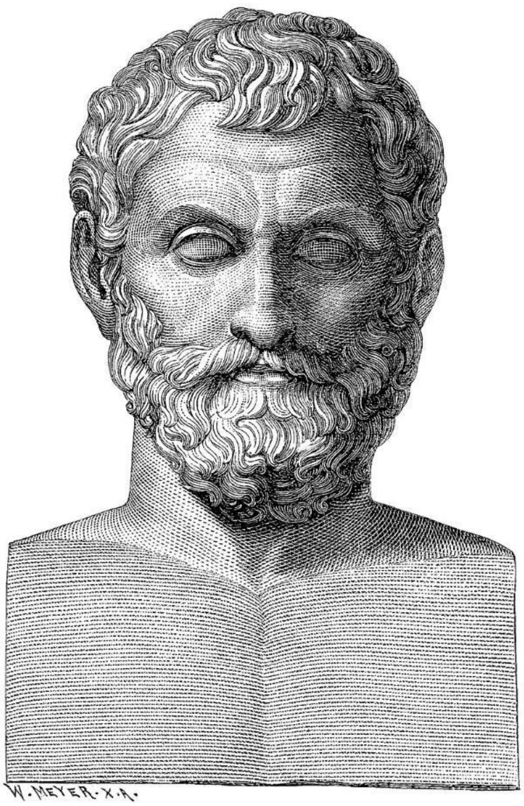
Respective blueprint	Sextant
Description	In this pedagogical sequence students will learn how to measure the heights of buildings using the Sextant and Thales's theorem on similar triangles
Learning Objectives	Students will: - Learn how to create a sextant - Understand the theorem of Thales on similar triangles - Learn how Xenagoras measured of the heights of mountains - Be able to measure the height of a building

Related curricular subject(s)	Geometry, History
Prerequisites / preparatory actions for teachers	Teachers should gather the materials for the blueprint
Prerequisites / preparatory actions for students	Understand the basics of geometry, know how to measure
Age of students	14-17
Duration	2-3 hours
Level of difficulty	Difficult

Step by step description of the tasks

Step 1: Who is Thales of Miletus?

The teacher introduces Thales to the class.



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Thales of Miletus (c. 624/623 – c. 548/545 BC) was a Greek mathematician, astronomer and pre-Socratic philosopher from Miletus in Ionia, Asia Minor. He was one of the Seven Sages of Greece. Many, most notably Aristotle, regarded him as the first philosopher in the Greek tradition, and he is otherwise historically recognized as the first

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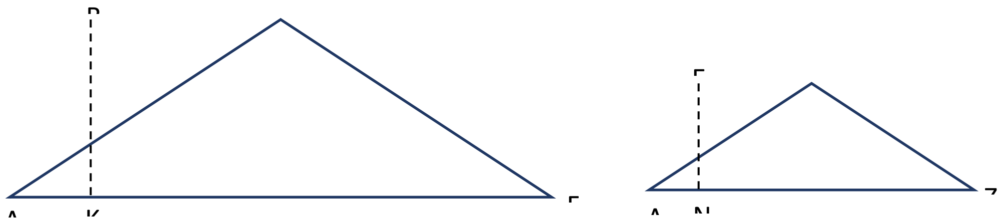
individual known to have entertained and engaged in scientific philosophy. He is often referred to as the Father of Science.

Step 2: Similar triangles theorem

The teacher explains Thales’ theorem on similar triangles as follows:

Two triangles are similar when all their respective angles are equal. According to Thales' theorem, similar triangles will have corresponding sides.

Let's look at the following example: Assume that the triangles ΑΒΓ and ΔΕΖ are similar and have sides ΑΒ= 4cm and ΔΕ = 2cm. The ratio of similarity λ, of ΑΒΓ to ΔΕΖ will be equal to 2. This means that the sides of the ΑΒΓ are twice as long as the sides of the triangle ΔΕΖ.



Thales' theorem states that $\frac{AB}{\Delta E}=\frac{A\Gamma}{\Delta Z}=\frac{B\Gamma}{EZ}=\lambda$ and also $\frac{BK}{EN}=\lambda$ where ΒΚ, ΕΝ is the height of the triangles.

That is, if we know that ΑΓ = 10cm then $\frac{A\Gamma}{\Delta Z}=2$ so, ΔΖ=5cm.

Step 3: Introduce and/or build the sextant

The teacher explains the utility of the sextant and directs the students to search for information at various places, eg.
https://en.wikipedia.org/wiki/Sextant

If the teacher wants, they can use the blueprint to build a sextant with the students.

Step 4: Presenting Xenagoras' experiment

The teacher should briefly explain the use of sextant to calculate the height of a mountain by Xenagoras and then the students can search for more information if the teacher wants to.

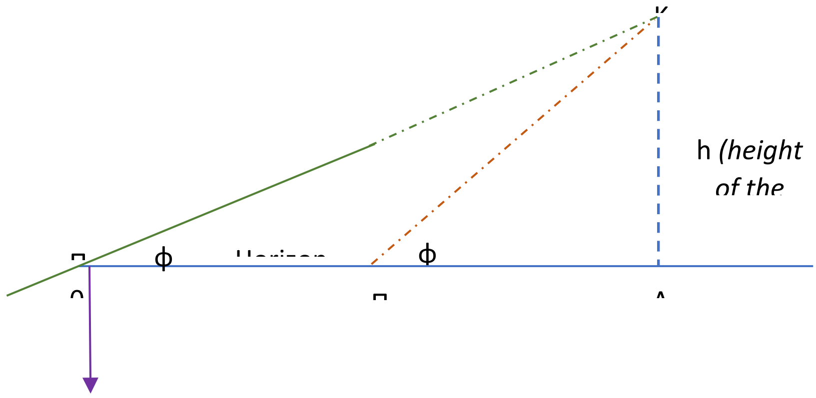
The first scientific study of Xenagoras (2nd century BC) was based on the theorems of Thales. He calculated the height of the peak of the Greek Western Olympus mountain, named Flambouros. Xenagoras used a kind of "diopter" to measure the altitude differences between this peak and the point of the ancient temple of Pythian Apollo where he was located. There, in the ancient temple at the foot of Olympus, he calculated the height of the peak to be 2479m. The exact height is 2473m as measured with modern tools. So, the deviation was only 6m. This experiment is saved through the texts of Plutarch.

Step 5: Measuring the height of the school

The teacher should briefly explain theoretically how they can measure the height of a building.

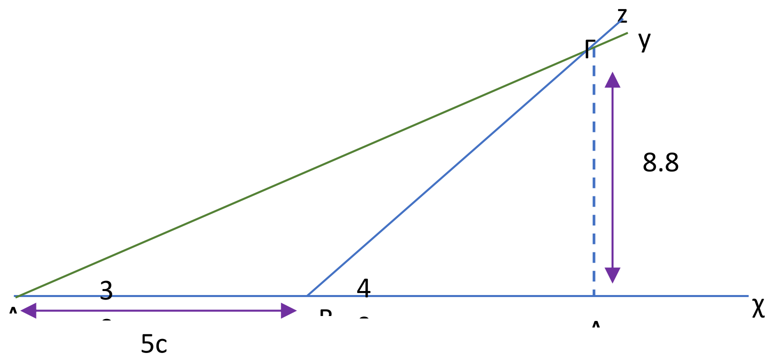
Using this improvised object an observer Π will aim at the top K of the building and with this construction we will find the angle φ under which shows the top of the building.

If we aim at point K from a point Π and the angle θ (formed by the fishing line and the wood) is, for example, 60° then the angle φ will be 30°, since the angle formed by the fishing line and the horizon will be a right angle, i.e. 90°.



We repeat the calculation in another Π΄ position. We measure the distance ΠΠ΄. Let us suppose that the distance ΠΠ΄ = 5m and the angle φ΄= 40°.

Having the above measurements, we make a model on a piece of paper. We bring a line Αχ and on it we get point B so that AB = 5cm. We then form the angles $\chi\hat{A}y = 30^o$ and $\chi\hat{B}z = 40^o$ and Γ is the point where Ay and Bz intersect.



From Γ we design a vertical line and measure it with a ruler. We find this to be 8.8cm. The triangles in the model are similar to the corresponding triangles in reality, so from Thales' theorem they will have their corresponding sides analogous.

This means that the ratio of the height h of the building to the height of the ΔΓ in the model, is equal to the ratio $\lambda = \frac{\Pi\Pi'}{\mathrm{AB}} = \frac{500cm}{5cm} = 100$. So the height of ΚΛ is 880 cm = 8.8 m. We need to add the height of the observer to this height to find the height of the school. For instance, if the observer is 1.80 then the height of the school will be 10.6 m.

Conclusion

In this lesson the students learn about the use of the sextant and Thales’ theorem. They learn how they were used in ancient Greece and they can use them to calculate the height of the building.

Assessment activities

Activity 1. Find information about Thales' works and present them to the class.

Activity 2. Search for information on how Xenagoras measured the peak of the mountain.

Activity 3. Measure the height of your school or of a tall tree in your school.